Name of the College. - S.S. College J-Bad.

Deptt - Mathematics.

Topic - Problem on Area of plane Curve (Contd.)

Class - B.Sc I (Hons

Time - 10.15 A.M To 11.00 A.M

11.00 A.M To 11.45 A.M

Date - 25-09-2020

By - Dr. Shree Nath Sharma

* Find the area between the curve.

$$x(x^2+y^2) = a(x^2-y^2)$$

Solution → Here Equation of the Curve

$$x(x^2+y^2) = a^2(x^2-y^2)$$

$$\Rightarrow x^3 + xy^2 = a^2x^2 - a^2y^2$$

Since Equation of Curve only Contains the even power of y ⇒ Curve is symmetrical about x-axis.

Since the Equation of Curve does not contain Const. Term ⇒ it passes through origin.

On Equating $x^2 - y^2 = 0$

$$\Rightarrow y = \pm x$$

Equating the co-efficient of highest power of $y = 0$,

$$a + x = 0$$

⇒ So $x = -a$ is the asymptote of the curve.

When $x = 0$, $y = 0$

When $x = a$ ⇒ $y = 0$

So the curve intersect x-axis at $(0,0)$ and $(a,0)$

The Curve is

$$x(x^2+y^2) = a(x^2-y^2)$$

$$\Rightarrow y^2(x+a) = (a-x)x^2$$

$$\Rightarrow \frac{y^2}{x^2} = \frac{a-x}{a+x}$$

$$\Rightarrow \frac{y}{x} = \pm\sqrt{\frac{a-x}{a+x}}$$

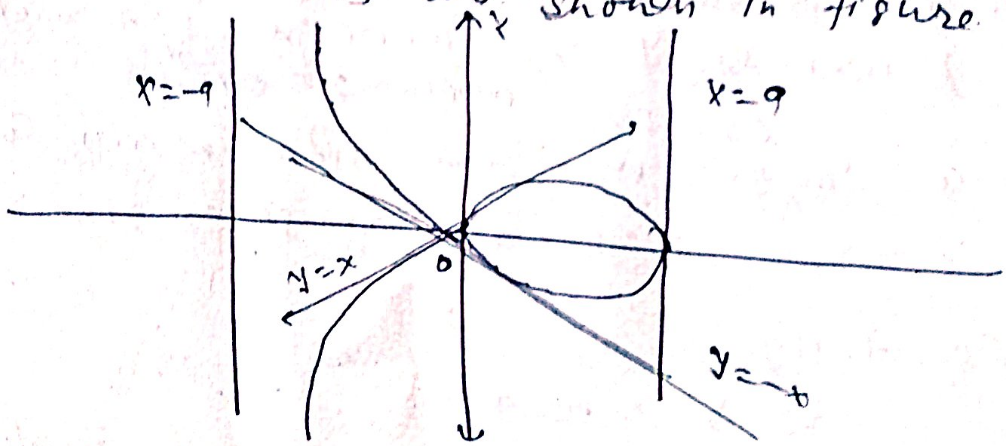
$$\therefore y = \pm x\sqrt{\frac{a-x}{a+x}}.$$

If $x > a$ y becomes imaginary

$\Rightarrow$ No part of the curve is in the right of $(a,0)$

The loop is situated between $x=0$ and $x=a$

and the curve is as shown in figure.



Therefore the area between the curve and its asymptote

$$= 2\int_{a}^{0} y\,dx = 2\int_{-a}^{0} -x\sqrt{\frac{a-x}{a+x}}\,dx$$

$$= -2\int_{-a}^{0} \frac{x(a-x)}{\sqrt{a^2-x^2}}\,dx$$

(-ve sign because this portion is in left of

Putting $x = a\sin\theta$

$\Rightarrow \theta = \sin^{-1}\frac{x}{a}$

When $x = 0$ $\theta = \sin^{-1}0 = 0$

When $x = +a$ $\theta = \sin^{-1}(-1) = \pi/2$

$$\therefore \text{Area of } = -2\int_{\pi/2}^{0} \frac{-a\sin\theta\,(a+a\sin\theta)}{\sqrt{a^2-a^2\sin^2\theta}}(-a\cos\theta)\,d\theta$$

$$= + 2a^2\int_{\pi/2}^{0}(\sin\theta + \sin^2\theta)\,d\theta.$$

$$= 2a^2\int_0^{\pi/2}[\sin\theta + \sin^2\theta]\,d\theta$$

$$= 2a^2\left\{\int_0^{\pi/2}\sin\theta\,d\theta + \int_0^{\pi/2}\sin^2\theta\,d\theta\right\}$$

$$= 2a^2\left[[-\cos\theta]_0^{\pi/2} + \frac{1}{2}\int_0^{\pi/2}(1-\cos 2\theta)\,d\theta\right.$$

$$= 2a^2\left[-(0-1) + \frac{1}{2}\left[\theta - \sin 2\theta\right]_0^{\pi/2}\right.$$

$$= 2a^2\{ + \frac{1}{2}a^2\left[\left(\frac{\pi}{2} - \sin\pi\right) - (0-\sin\pi)\right]$$

$$= 2a^2 + \cancel{b} \times \frac{\pi}{2}a^2$$

$$= a^2\left(2+\frac{\pi}{4}\right)$$

$$= \frac{4+\pi}{a^2}$$

the Area of the loop

$$= 2\int_0^a y\,dx = 2\int_0^a x\sqrt{\frac{a-x}{a+x}}\,dx$$

$$= 2\int_0^a \frac{x(a-x)}{\sqrt{a^2-x^2}}\,dx$$

Putting $x = a\sin\theta$ $\quad dx = a\cos\theta\,d\theta$.

$\sqrt{a^2-x^2} = a\cos\theta$.

When $x=0$ $\quad \theta = 0$

When $x=a$. $\quad \theta = \pi/2$

$$\therefore \text{Area} = 2\int_0^{\pi/2} \frac{a^2\sin\theta(1-\sin\theta)\,a\cos\theta\,d\theta}{a\cancel{\cos\theta}}$$

$$= 2a^2\int_0^{\pi/2}(\sin\theta - \sin^2\theta)\,d\theta$$

$$= 2a^2\int_0^{\pi/2}\left(\sin\theta - \frac{1-\cos 2\theta}{2}\right)d\theta$$

$$\cancel{= 2a^2\int_0^{\pi/2}[2\sin\theta + \cos 2\theta - 1]\,d\theta}$$

$$\cancel{= 2a\left[-2\cos\theta + \frac{\sin 2\theta}{2} - \theta\right]_0^{\pi/2}}$$

$$= 2a^2\left\{\left\{\cancel{-2\cos\pi/2} + \frac{a\sin 2\pi/2}{2} - \pi/2\right\} - \left\{-2\cos 0 + 0 - 0\right\}\right\}$$

$$= 2a^2\left\{\left[-0 + 0 - \pi/2\right] - (-2)\right] = 2a^2\left(1 - \frac{\pi}{4}\right)$$

$$= 2a^2 \int_0^{\pi/2} \left[\sin\theta - \left(\frac{1+\cos 2\theta}{2}\right)\right] d\theta$$

$$= 2a^2 \left[-\cos\theta - \frac{1}{2}\theta + \frac{\sin 2\theta}{4}\right]_0^{\pi/2}$$

$$= 2a^2 \left[\frac{\sin 2\theta}{4} - \cos\theta - \frac{\theta}{2}\right]_0^{\pi/2}$$

$$= 2a^2 \left[\left\{\frac{\sin\pi}{4} - \cos\pi/2 - \frac{\pi}{4}\right\} - \{0 - 1 - 0\}\right]$$

$$= 2a^2 \left[0 - 0 - \frac{\pi}{4} + 1\right]$$

$$= 2a^2 \left[1 - \frac{\pi}{4}\right] = 2a^2\left(\frac{4-\pi}{4}\right)$$

$$= \frac{a^2(4-\pi)}{2} \text{ Sq unit.}$$